

Optimal Filter Design with Parametrized Wavelets

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We first outline the construction of parametrized families of orthonormal wavelets with several vanishing moments. Using Symbolic Computation and in particular Gröber bases, we obtain symbolic expressions for the corresponding filter coefficients in terms of the first discrete moments. Then we discuss the regularity of wavelets and scaling functions. The Sobolev regularity can be computed from the spectral radius of a matrix depending on the filter coefficients, and there are methods to compute lower bounds for the Hölder regularity. The symbolic parametrization of filter coefficients allows us to compute the most regular wavelet for a fixed number of filter coefficients by (simple) numerical methods.

It is well known that the only symmetric and orthonormal wavelet is the Haar wavelet. Symmetry of the scaling function is in turn equivalent to symmetry of the filter coefficients. Using the parametrized filter coefficients, we illustrate how we can construct the least asymmetric filters by computer algebra and give several examples. We also discuss the use of parametrized wavelets for compression by computing an optimal parameter for a given signal. Finally, we demonstrate a GUI and several MATLAB functions to compute with and illustrate parametrized wavelets.