Solution of Nonlocal Cauchy Problems by Extensions of Duhamel Principle and Heaviside Algorithm

Ivan Dimovski, Margarita Spiridonova

Institute of Mathematics and Informatics,
Bulgarian Academy od Sciences

E-mail addresses: dimovski@math.bas.bg, mspirid@math.bas.bg

We consider nonlocal Cauchy problems consisting in solution of linear differential equations of the form

$$P(\frac{d}{dt})\, y \;=\; F(t)$$

under the "initial" conditions

$$\Phi\{\, y^{(k)}\} \;=\; 0,\;\; k \;=\; 1,\, 1,\, 2,\, \ldots,\, degP - 1,$$

where $P(\lambda)$ is a polynomial with constant coefficients, $F(t) \in C(I\!R)$ and Φ is a linear functional in $C(I\!R)$.

The Duhamel principle is extended in the following way:

Let Y= Y(t) be the solution of the nonlocal Cauchy problem for $F(t) \;\equiv\; 1$. Then the solution for arbitrary $F(t)$ is given by

$$y(t) \;=\; \frac{d}{dt}\,\Phi_\tau\,\{\int_\tau^t Y(t+\tau-\sigma)\,F(\sigma)\,d\sigma\,\}.$$

The convolution

$$(f*g)(t) \;=\; \Phi_\tau\,\{\int_\tau^t f(t+\tau-\sigma)\,g(\sigma)\,d\sigma\,\},$$

introduced by one of the authors in 1974 (see [1]) allows to build a Mikusinski-type operational calculus based on it and to extend the Heaviside algorithm (see [2]) for obtaining the special solution $Y \;=\; Y(t)$.

The specialization to the functional $\Phi\{f\} = \frac{1}{T}\int_0^T f(\tau)\,d\tau$ allows to propose an efficient algorithm for obtaining of the periodic solutions of linear ordinary differential equations with constant coefficients both in the non-resonance and in the resonance cases (see [3]). This algorithm was implemented and experimented in the environment of the computer algebra system *Mathematica*.

References. [1] I. H. Dimovski. Convolutional Calculus. Kluwer, Dordrecht, 1990. [2] I. H. Dimovski. Nonlocal operational calculi. In Proc. Steklov Inst. of Math., 1995, Issue 3, 53-65. [3] S. I. Grozdev. A convolutional approach to initial value problems for equations with right invertible operators. Compt. Rend., Bulg. Acad. of Sci., 33, 1 (1980), 35-38.