Data processing and refinement

P. Schapotschnikow

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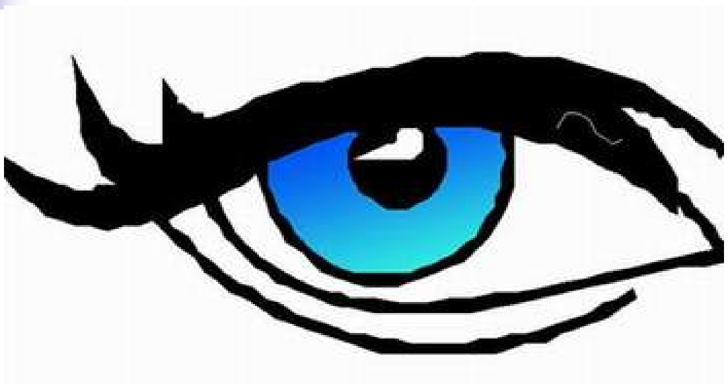
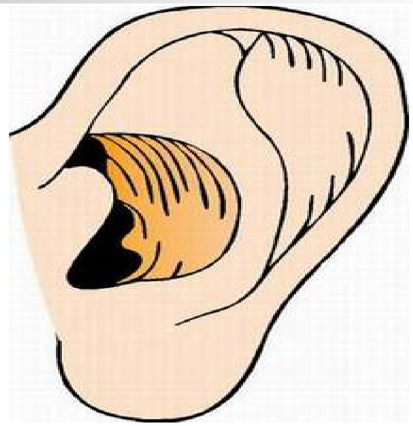
Vision vs Hearing





- High 2D resolution
- 3D reconstruction
- Different receptors allow colour vision

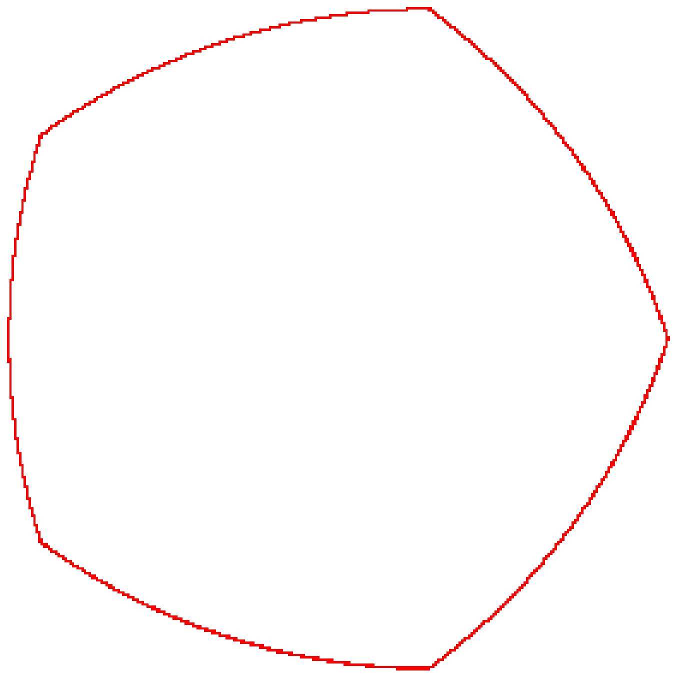
- Low spatial resolution
- Reconstruction of different sounds in a mixture

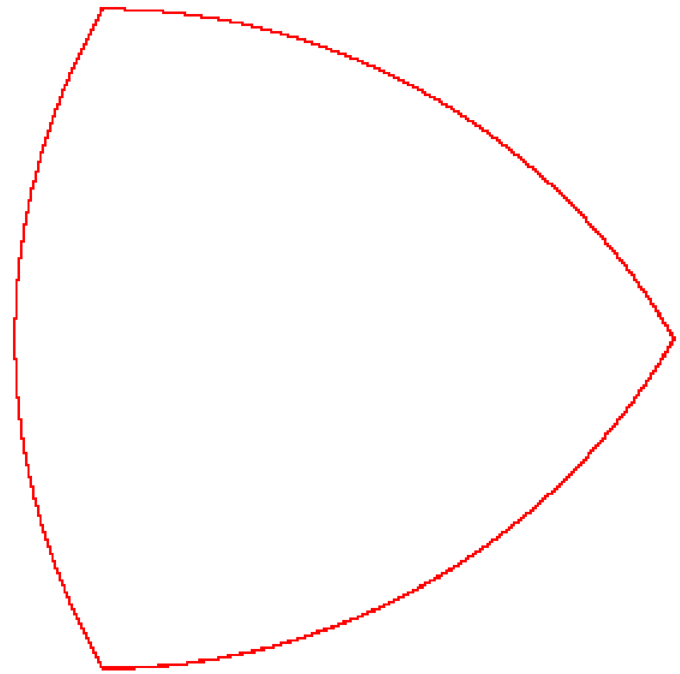
2D reconstruction from silhouettes

- Is it possible to reconstruct a convex object given all 2D parallel-beam silhouettes?

2D reconstruction from silhouettes

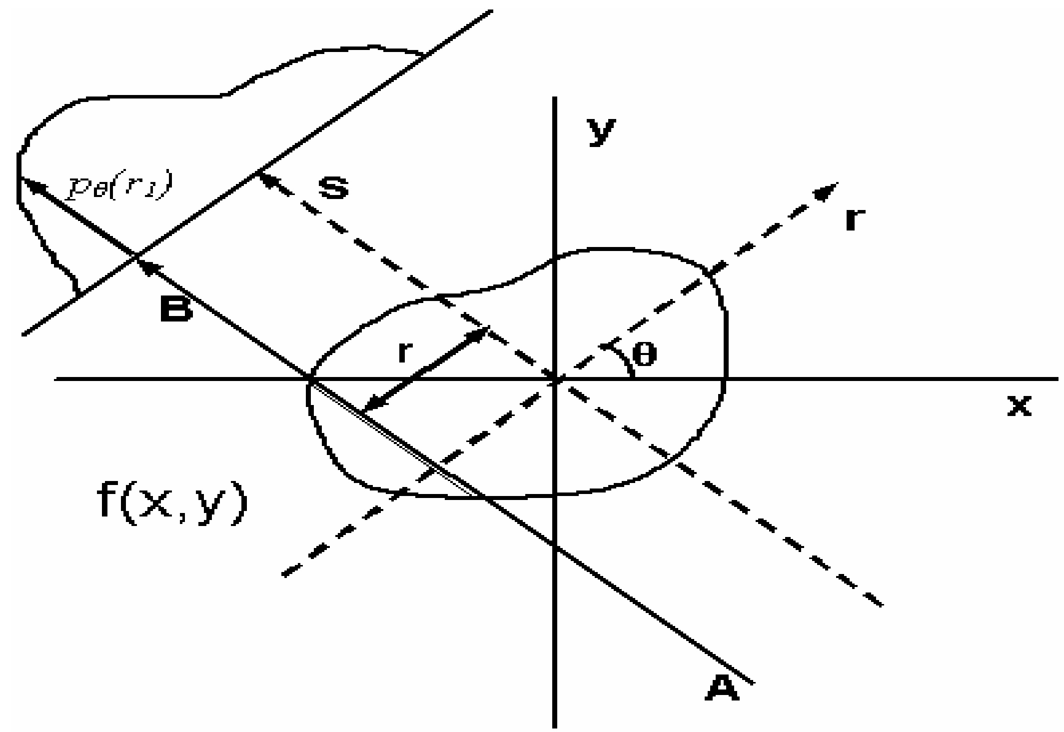
NOT UNIQUELY!!





Tomography reconstruction

CT: reconstruction from X-Ray shadows



Tomography reconstruction

- Intensity of shadows: Beer-Lambert law

$$I = I_0 \exp(-\int \mu(x)dx)$$

- μ(x) is the attenuation coefficient varying for different tissues
- Silhouettes are special cases with infinite attenuation

Tomography reconstruction

- Radon transform

$$\ln \frac{I_0}{I} = \int_{l(r,\theta)} \mu(\vec{x})\, dx = p(r,\theta)$$

- Inverse Radon transform

$$\mu(\vec{x}) = \frac{1}{2\pi^2} \int_0^{\pi} \int_{-\infty}^{\infty} \frac{1}{x_1 \cos\theta + x_2 \sin\theta - r} \frac{\partial p(r,\theta)}{\partial r}\, dr\, d\theta$$

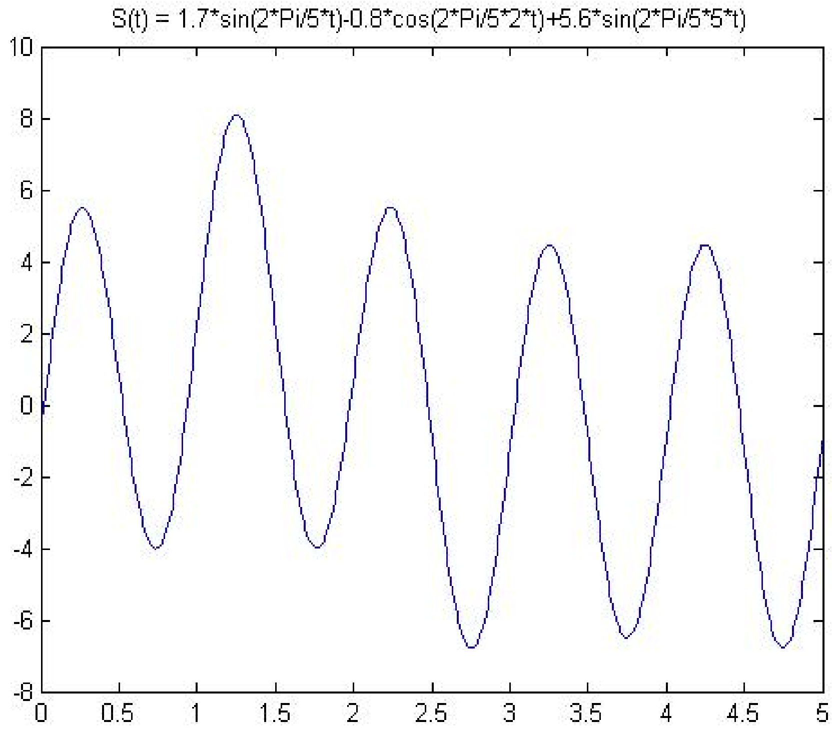
Tomography reconstruction

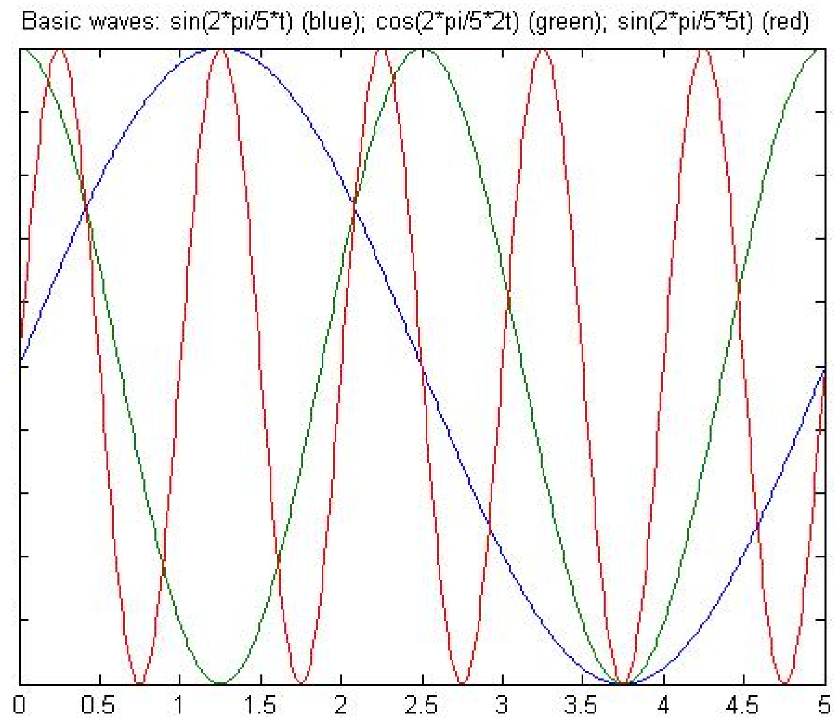
- Problem: the iRT is discontinuous!

→ Filtered Back-Projection

Fourier Transform

- Task: find the share of a basic signal in a mixture





Fourier Transform

- Trick: integrate with the required mode

$$\int_0^T S(t)\sin(\frac{2\pi}{T}t)dt =$$

$$\int_0^T b_1 \sin^2(\frac{2\pi}{T}t)dt + \int_0^T a_2 \sin(\frac{2\pi}{T}t)\cos(\frac{2\pi}{T}2x)dt + \int_0^T b_5 \sin(\frac{2\pi}{T}t)\sin(\frac{2\pi}{T}5x)dt =$$

$$\frac{T}{2}b_1 + 0 + 0.$$

Fourier Transform

- Every signal can be split in frequencies k/T

- Frequency resolution is determined by the time range

Fourier Transform

- Complexification: deMoivre formula

$$\exp(ix) = \cos(x) + i\sin(x)$$

$$\Downarrow$$

$$a\cos(\omega x) + b\sin(\omega x) = c_+ \exp(i\omega x) + c_- \exp(-i\omega x)$$

$$a = c_+ + c_- \qquad b = i(c_+ - c_-).$$

Fourier Transform

- Discretise the time domain:

$$T \longrightarrow N\Delta t$$

$$\Downarrow$$

$$\text{Integral } dt \longrightarrow \text{sum } \Delta t$$

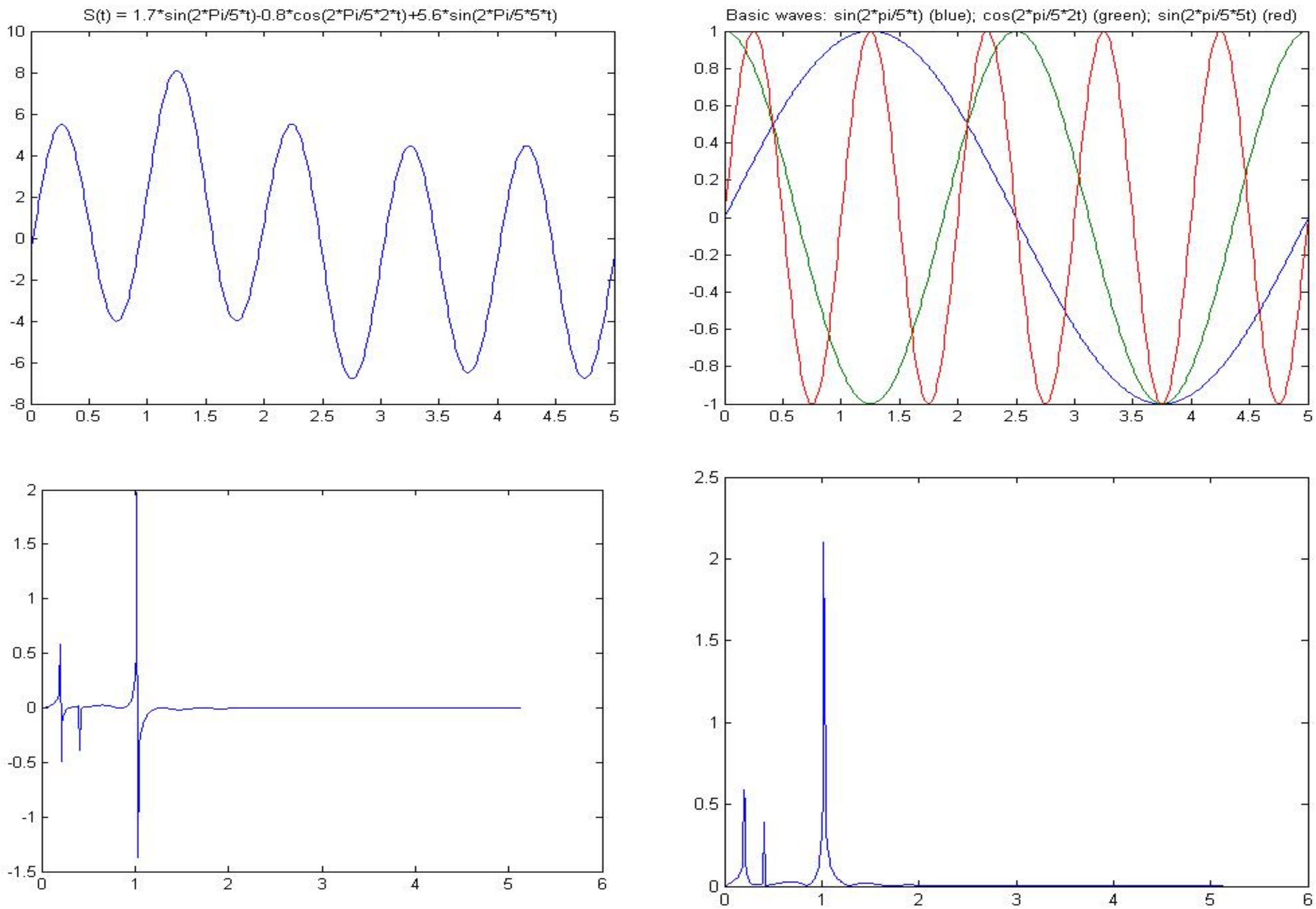
Fourier Transform

- Discrete Fourier Transform

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} \exp(-i 2\pi k n / N) R_n$$

- Aliasing
- Redundancy

Fourier Transform



Fourier Transform

- Uncertainty
  - Bandwidth

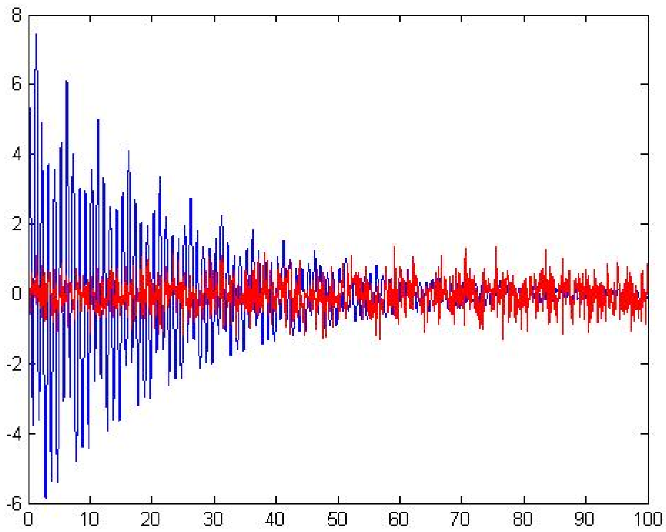
$$\frac{N-1}{2}\cdot\frac{1}{N\Delta t}\approx\frac{1}{2\Delta t}$$

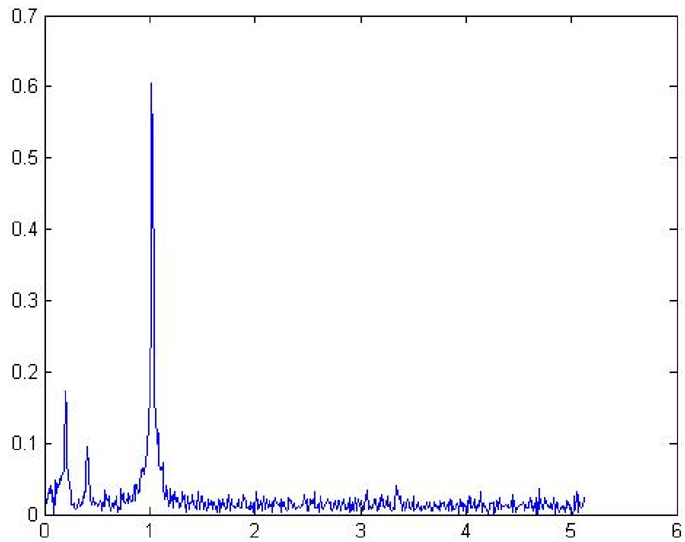
  - Time/Frequency resolution

$$\Delta t\Delta f=\frac{1}{N}$$

Fourier Transform

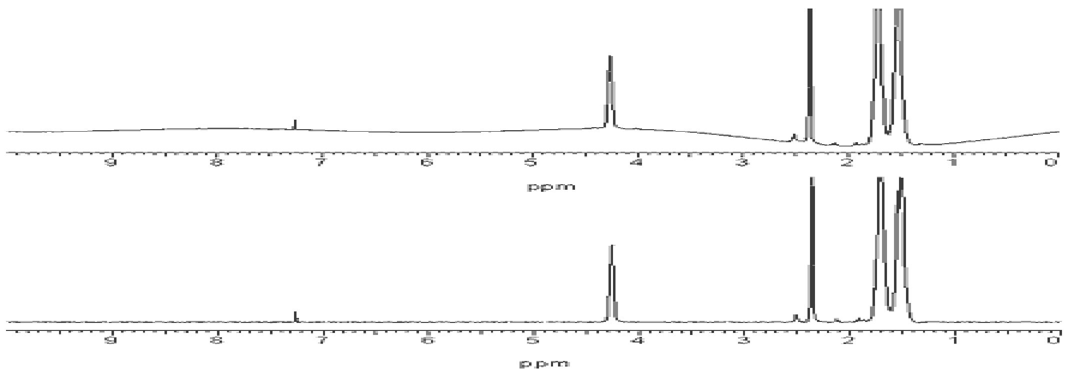
- NMR-signals:
  - exponential decay
  - noise





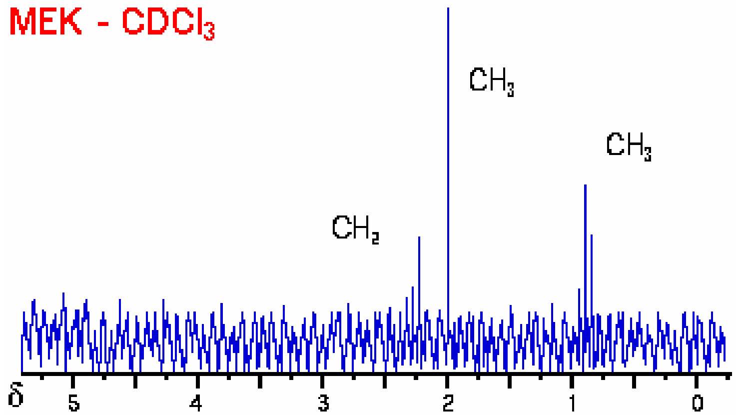
Data refinement

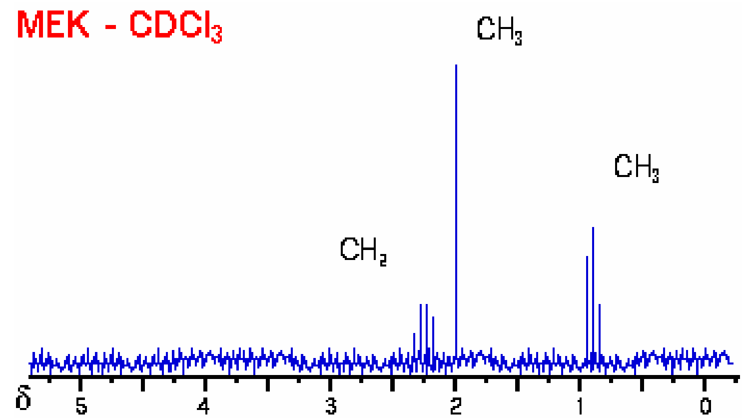
- Baseline correction



- First point extrapolation
- Adopization (weighting)
- Zero filling

Averaging


MEK - $CDCl_3$
CH_3
CH_3
CH_2
δ 5 4 3 2 1 0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CH_3
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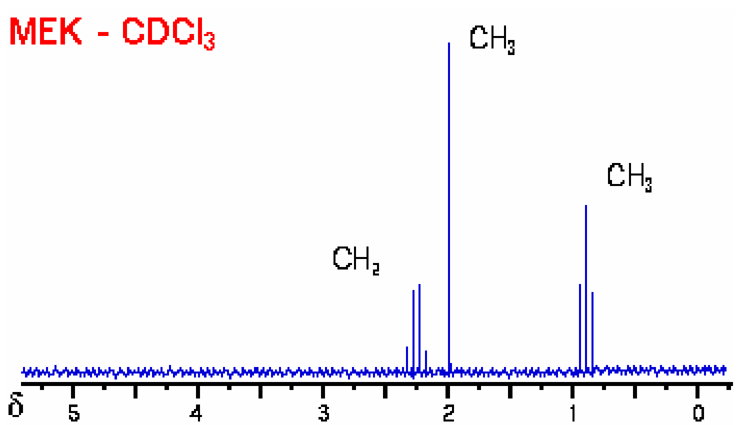
δ 5 4 3 2 1 0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CH_3
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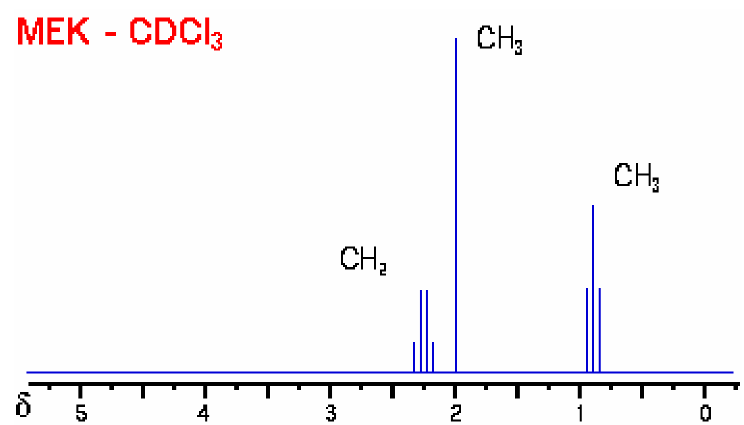
δ 5 4 3 2 1 0


MEK - $CDCl_3$
CH_3
CH_3
CH_2
δ 5 4 3 2 1 0

THANK YOU FOR YOUR ATTENTION!