Numerical Simulation Of Noise Generation And Propagation In Turbo Machinery

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1. Introduction
2. Acoustic Models
   - The Lighthill Analogy
   - Expansion About Incompressible Flow By Hardin & Pope
3. Numerical Simulations
   - Overview
   - Radial Pump
   - Axial Pump
4. Conclusion

AEROACOUSTICS

AEROACOUSTICS

The field of Aeroacoustics is that part of fluid dynamics, where sound generation and propagation in a moving medium are studied.

Definition of sound

- Sound is represented as density disturbances in the flow field
- Disturbances propagate as waves over large distances through a medium like fluids, gases or solids
- Sound waves can be perceived by the hearing sense of a human being

Sound and Noise

Sound is a change in pressure with respect to atmosphere, whereas noise is unwanted sound.

Direct numerical simulation

One possibility for computation is the direct approach:

- Direct numerical simulation of complete compressible Navier-Stokes equations requires immense computational effort Go to DNS
- Extreme fine grid resolution because of great differences in length and energy scales between hydrodynamic and acoustic field

⇒ Development of **hybrid methods**, which split noise calculation into

1. computation of the flow field
2. computation of the acoustic field

HYBRID APPROACH

- Splits the computational domain into two parts, the governing flow field and the acoustic field
- Enables usage of different numerical solvers
  - dedicated CFD tool
  - acoustic solver
- The solution of the flow field is then used as input for the second solver, which calculates the acoustical propagation

CONSIDERATION OF TWO HYBRID METHODS

This talk introduces two different hybrid methods:

- An integral method based on the **Lighthill analogy**
- **Expansion about Incompressible Flow**, developed by Hardin & Pope

Consideration of two hybrid methods

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MULTIPOLE SOURCES

Noise, as a result of flow interacting with rotating surfaces, is described by 3 kinds of sources

1. **Monopoles**: thickness noise, i.e. the surface distribution due to the volume displacement of fluid during the motion of the surfaces
2. **Dipoles**: loading noise, i.e. surface distribution due to the interaction of flow with moving bodies
3. **Quadrupoles**: vortex noise, i.e. field distribution due to flow outside the surfaces

Lighthill's point of departure

- Lighthill reformulated the compressible Navier-Stokes equations
- He derived a linear wave equation with a quadrupole-like source term, which includes a pressure and density contribution

Lighthill analogy

According to the Lighthill analogy, the noise due to an unsteady flow is equivalent to the noise, that is generated by equivalent quadrupole sources radiating in a medium at rest!

Extensions to the Lighthill model

1. **Curle**
   - Extension in order to handle the influence of solid boundaries upon aerodynamic sound
   - Noise, due to a flow passing by a body, is equivalent to noise generated by dipoles on the surfaces and quadrupoles outside the surfaces
2. **Ffowcs-Williams and Hawkings**
   - Extension to handle the interaction of flow with rotating surfaces
   - Introduction of mathematical surfaces, that coincide with surfaces of the moving solid, and imposing boundary conditions on it

Derivation of the acoustic equation Part I

Consider the compressible Navier-Stokes equations (without energy equation)

- conservation of mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = q$$

- conservation of momentum

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + f_i$$

Derivation of the acoustic equation Part II

- By
  - differentiating the continuity equation with respect to time
  - differentiating the conservation of momentum equation with respect to x_i
  - subtract the latter from the first
- One gets the **acoustic wave equation**

$$\frac{\partial^2 \rho}{\partial t^2} - \frac{\partial^2 p}{\partial x_i^2} = \frac{\partial q}{\partial t} - \frac{\partial f_i}{\partial x_i} + \frac{\partial^2}{\partial x_i \partial x_j}\left(\rho u_i u_j\right)$$

- Using a perturbation ansatz for linearisation

$$\begin{aligned} \rho &= \rho_0 + \rho' \\ p &= p_0 + p' \end{aligned}$$

Lighthill's acoustic equation

$$\frac{\partial^2 \rho'}{\partial t^2} - a_s^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \underbrace{\frac{\partial q}{\partial t}}_{(1)} - \underbrace{\frac{\partial f_i}{\partial x_i}}_{(2)} + \underbrace{\frac{\partial^2 \tau_{ij}}{\partial x_i \partial x_j}}_{(3)}$$

1. Monopoles
2. Dipoles
3. Quadrupoles

The situation of a flow interacting with a rotating surface is equivalent to an acoustic medium at rest containing 3 source distributions!

Go to Derivation

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THE HYDRODYNAMIC EQUATIONS

- Consider viscous flow with $Ma \leq 0.3$
- Motion of fluid is described by **compressible Navier-Stokes equations**

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} &= 0 \\
\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j + p_{ij} - f_{viscous})}{\partial x_j} &= 0 \\
p &= p(\rho, S) \\
\rho T \frac{DS}{Dt} = \rho c_p \frac{DT}{Dt} - \beta T \frac{Dp}{Dt} &= \rho \phi + \frac{\partial}{\partial x_i}\left(k \frac{\partial T}{\partial x_i}\right)
\end{aligned}
$$

THE INCOMPRESSIBLE SOLUTION

- For EIF it is assumed that the fluid flow is at rest (constant density ρ_0 and constant pressure p_0)
- The incompressible solution is obtained by the **incompressible Navier-Stokes equations**

$$\begin{aligned}
\frac{\partial U_i}{\partial x_i} &= 0, \\
\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} &= -\frac{1}{\rho_0}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j^2}
\end{aligned}$$

P	:	incompressible pressure
ρ_0	:	incompressible density
U_i	:	velocity components

PRESSURE CHANGE

- The pressure change from ambient pressure p_0 is given by

$$\begin{aligned} dp &= P - p_0 \\ \Rightarrow dp &= \left(\frac{\partial p}{\partial \rho}\right)_S d\rho + \left(\frac{\partial p}{\partial S}\right)_\rho dS = a_s^2 d\rho + \left(\frac{\partial p}{\partial S}\right)_\rho dS \end{aligned}$$

- Speed of sound: $a_s = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_S}$

Eliminating the energy equation

- Introduce time-averaged incompr. pressure distribution

$$\bar{P}(x) = \lim_{T\to\infty} \frac{1}{T}\int_0^T P(x,t)dt$$

- Under the assumption of isentropic pressure fluctuations in time $p - \bar{P}$, it is

$$p(\rho, S) = p^*(\rho) + \bar{P}(S)$$

- Therefore $\bar{P}$ involves losses, p^* is assumed to be isentropic

$$\frac{\partial p}{\partial t} = \frac{\partial p^*}{\partial t} = \frac{dp^*}{d\rho}\frac{\partial \rho}{\partial t} = \left(\frac{\partial p}{\partial \rho}\right)_S \frac{\partial \rho}{\partial t} = a_s^2 \frac{\partial \rho}{\partial t}$$

$\Rightarrow$ Because of the isentropic assumption there is no need for an energy equation in acoustic description

An approach by Hardin & Pope

- Hardin & Pope proposed a nonlinear two-step procedure
- Suitable for both noise generation and propagation
- Decouples the flow field into
  1. incompressible viscous flow field
  2. compressible inviscid acoustic field
- Correction to the constant hydrodynamic density is used

$\Rightarrow$ Acoustic radiation is obtained from numerical solution of a system of perturbed, compressible and inviscid equations

DECOMPOSITION OF THE COMPRESSIBLE SOLUTION

$$
\begin{aligned}
u_i(x_i,t) &= U_i(x_i,t) + u'_i(x_i,t)\\
p(x_i,t) &= P(x_i,t) + p'(x_i,t)\\
\underbrace{\rho(x_i,t)}_{(1)} &= \underbrace{\rho_0(x_i,t)}_{(2)} + \underbrace{\rho'(x_i,t)}_{(3)}
\end{aligned}
$$

1. Exact numerical solution of the fluiddynamic and acoustic problem
2. Unsteady flow variables without acoustic portion
3. Acoustic quantities, sound propagation

Note: For compressible flows with $Ma \leq 0.3$ it can be set $\rho_0 = \text{const}$.

Derivation of the acoustic equations

- Insert decomposition into hydrodynamic equations
- Neglect terms of viscosity on the fluctuations
- Obtain **first-order nonlinear system of acoustic equations** Go to Derivation

$$
\begin{aligned}
\frac{\partial \rho'}{\partial t} &+ \frac{\partial f_i}{\partial x_i} = 0 \\
\frac{\partial f_i}{\partial t} &+ \frac{\partial}{\partial x_j}\left[f_i(U_j + u_j') + \rho_0 U_i u_j' + p'\delta_{ij}\right] = 0 \\
\frac{\partial p'}{\partial t} &- a_s^2 \frac{\partial \rho'}{\partial t} = -\frac{\partial P}{\partial t} \quad \Leftrightarrow \quad \frac{\partial p'}{\partial t} + a_s^2 \frac{\partial f_i}{\partial x_i} = -\frac{\partial P}{\partial t}
\end{aligned}
$$

Note: $f_i = \rho \cdot u_i' + U_i \cdot \rho'$, and $a_s^2 = \gamma \frac{P+p'}{\rho_0+\rho'}$, with $\gamma = \frac{c_p}{c_v} = 1.4$

ALGORITHM

- Instantaneous pressure: single information which is coming from the incompressible solution

$\Rightarrow$ Acoustic calculation can be started at any time during incompressible computation

Fluid-Dynamic equations + initial- and boundary conditions

$$\downarrow \; P(x_i, t) \; \downarrow$$

Acoustic equations + initial- and boundary conditions

$$\downarrow$$

$$\boxed{\rho'(x_i, t), \quad p'(x_i, t)}$$

Initial conditions

- Appropriate initial conditions at the inlet are

$$\begin{aligned} \rho' &= 0 \\ u_i' &= 0 \quad \text{respectively } f_i = 0 \\ p' &= p_0 - P \end{aligned}$$

- p_0 denotes the constant ambient pressure

Boundary Conditions at Solid Walls

- **Remember:** the acoustic equations are inviscid

$\Rightarrow$ The only boundary condition at solid walls is the slip condition

$$\begin{aligned} u_n &= 0 \\ f_n &= 0 \end{aligned}$$

Acoustic boundary conditions

- Acoustic boundary conditions: non-reflecting boundary conditions at the borderline of the computational domain
- Radiation boundary conditions proposed by Tam & Webb
- **Aim:** reducing non-physical reflection from computational boundaries

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SOFTWARE TOOLS

- Simulation of sound propagation and generation is obtained with the following components

NS3D	:	hydrodynamic solver, developed at the **FLM** Go to NS3D
SYSNOISE	:	commercial tool for acoustics, based on aeroacoustic analogy by Lighthill
EIF	:	under development

- Interface between NS3D and SYSNOISE allows transmission of the transient solutions for pressure and velocity from NS3D to SYSNOISE

Simulation routine with SYSNOISE

Fluiddynamic solver NS3D

↓

Postprocessing and Convertion to SYSNOISE

↓

Import data into SYSNOISE

↓

Simulation and Visualization in SYSNOISE

TESTCASES

Two test cases will be presented:

- Radial Pump RP28
- Axial Pump AP149

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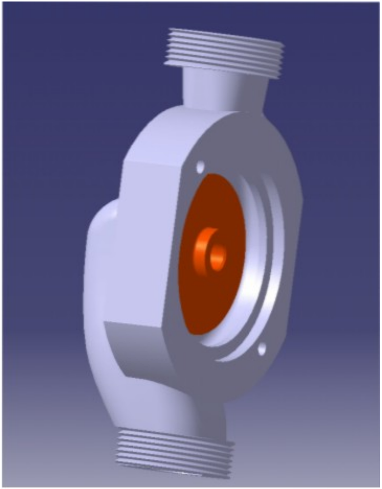
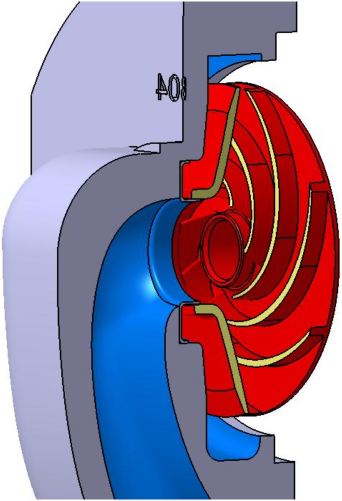
General informations

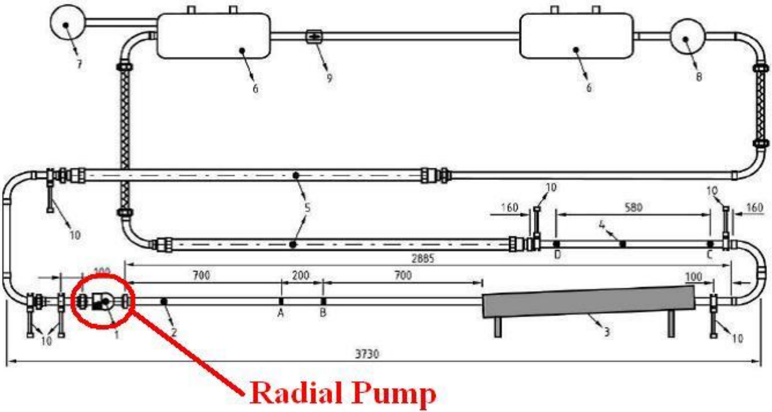
- Provided by **Wilo AG**
- Acoustic measurements for validation already exist
- Radial pump has an impeller with 7 blades
- Char. $Re = 6 \cdot 10^5$
- Analysis takes place at the following operating points
  - **Optimal load**: $Q = 2.5 \frac{m^3}{h},\ n = 2524 \frac{1}{min}$
  - **Partial load**: $Q = 1.0 \frac{m^3}{h},\ n = 2690 \frac{1}{min}$

Geometry of the radial pump

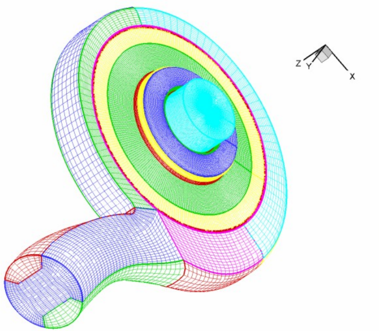




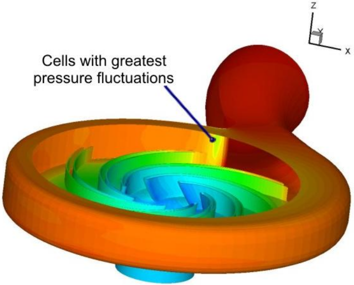
THE COMPLETE INSTALLATION



Point of greatest Pressure fluctuations

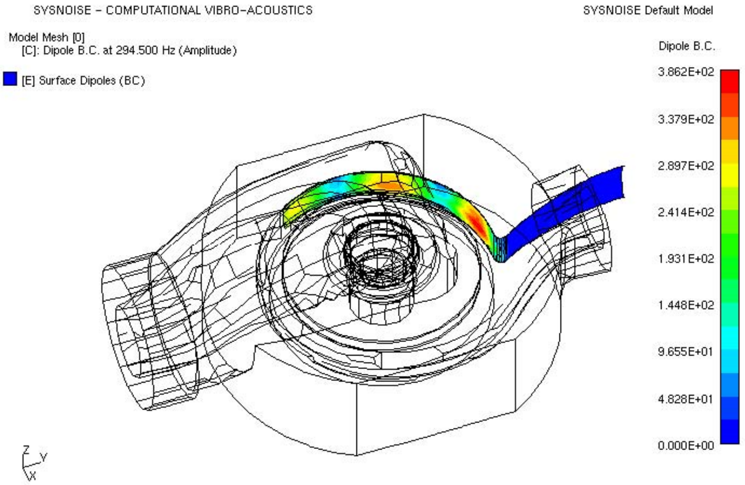


Computational grid for flow solution for impeller, spiral case and sidechamber

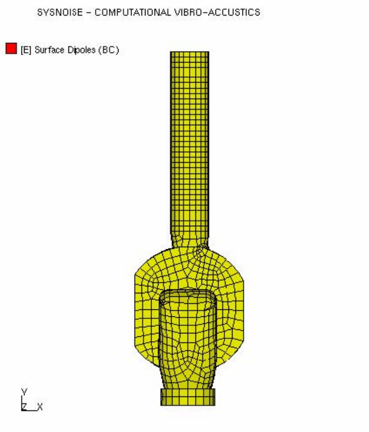


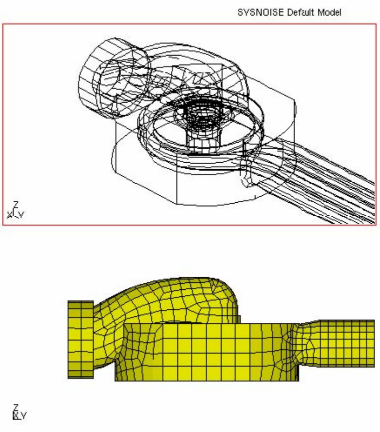
Pressure fluctuations in the pump

PRESSURE DISTRIBUTION



COMPUTATIONAL GRID FOR ACOUSTIC COMPUTATIONS

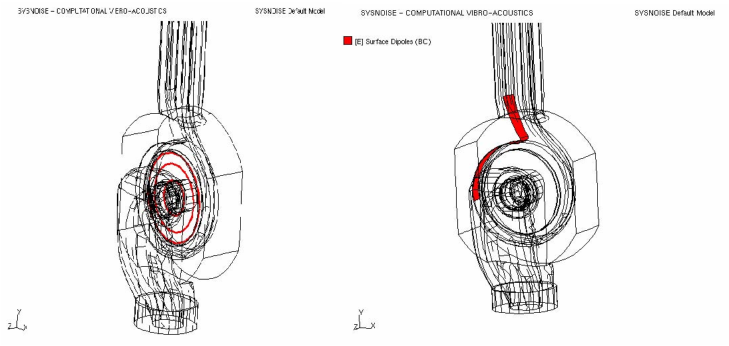




First step

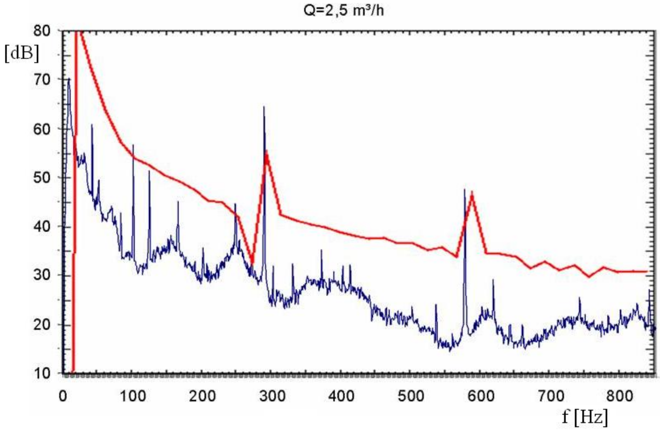
- Investigation of the sound generation, resulting from pressure fluctuations at the rotating blades
- Therefore
  - Definition of rotating dipole sources on the surface of the blades
  - Definition of distributed fixed dipole sources on the walls of the spiral case at the outlet area

DIPOLE SOURCES FOR FIRST STEP



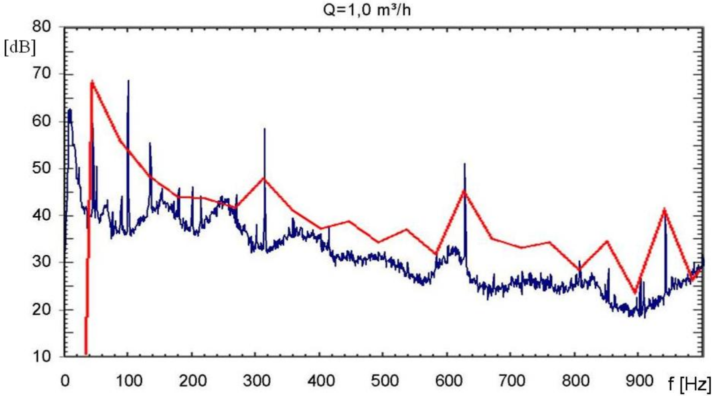
Definiton of the (LEFT) rotating dipole sources and (RIGHT) fixed dipole sources at the volute tongue

RESULTS FROM STEP ONE FOR FIXED DIPOLES



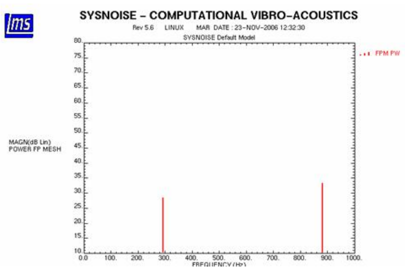
Flow-induced sound power through fixed dipoles under optimal load $Q = 2.5\ \frac{m^3}{h}$; blue measurement, red computation

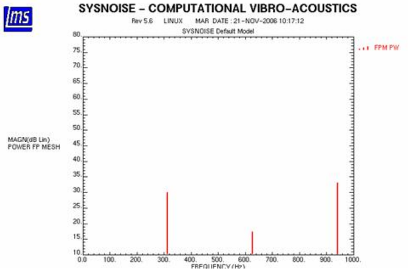
Results from step one for fixed dipoles



Flow-induced sound power through fixed dipoles under partial load $Q = 1.0\ \frac{m^3}{h}$; blue measurement, red computation

RESULTS FROM STEP ONE FOR ROTATING DIPOLES





Flow-induced sound power through rotating dipoles;

(LEFT) $Q = 2.5\ \frac{m^3}{h}$, (RIGHT) $Q = 1.0\ \frac{m^3}{h}$

Analysis of the results from step one

- In contrast to the experimental measurements, the resulting flow-induced sound power is too inaccurate

⇒ Considering of additional pressure fluctuations at the interior of the pump

⇒ **Second step:**

1. Define supplementary fixed dipoles in the volute tongue area (the area around the outlet)
2. Consider additional areas for definition of further source terms from the flow solution

Analysis of the results from step one

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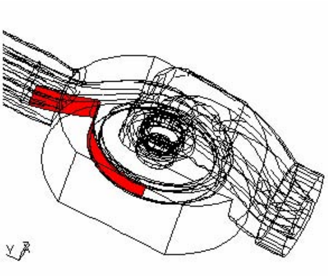
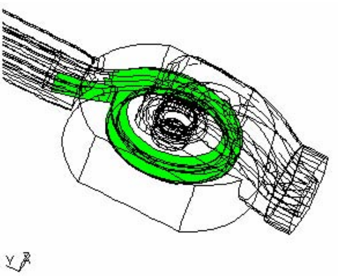
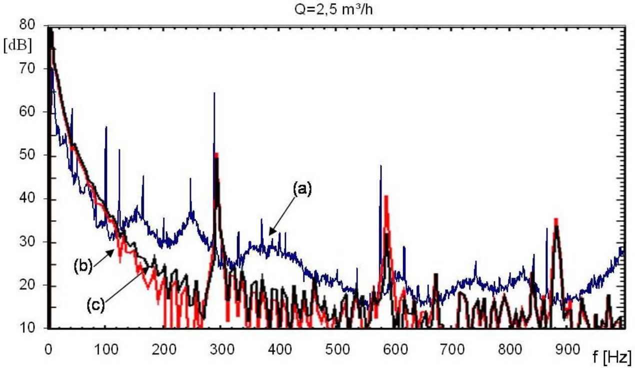
DISTRIBUTED DIPOLE SOURCES





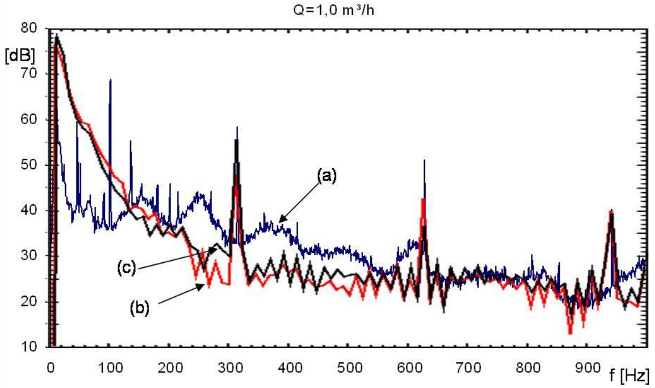
Definition of the distributed dipoles (red) volute tongue area and (green) extended area

Flow-induced acoustic power (optimal load)



Flow-induced acoustic power under optimal load (a) measurement (b) calculation with dipoles in the volute tongue area (c) calculation with dipoles in the extended area

Flow-induced acoustic power (partial load)



Flow-induced acoustic power under partial load (a) measurement (b) calculation with dipoles in the volute tongue area (c) calculation with dipoles in the extended area

Summary of the results

Comparison of the two results shows

- Dominating dipole sources are situated at the volute tongue of the pump
- Additional dipole sources in the extended area have nearly no influence on the computed acoustic power

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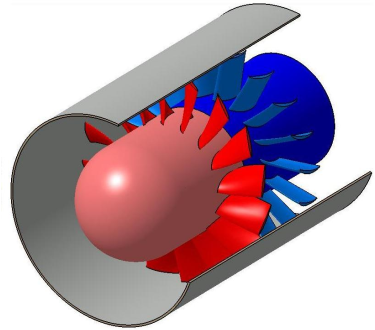
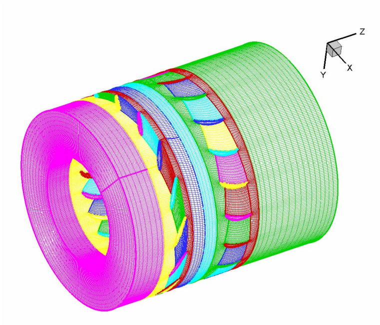
General informations

- Specific rotational frequency $n_q \approx 150$
- Number of rotor blades $Z_{rot} = 16$
- Number of stator blades $Z_{stat} = 19$
- Investigations at the TU Braunschweig for validation have already been made

Geometry and mesh for the axial pump



Geometry of the axial pump



Meshing of the axial pump

Analysis of the hydrodynamic equations

- Computation of the flow field shows:
  - The most intense pressure fluctuations arise at the point where stator-rotor interaction takes place, more precise where the unsteady flow hits the tip of the stator
  - I.e. at the hub

⇒ Consider only fixed dipoles at the stator as acoustic sources

Analysis of the hydrodynamic equations

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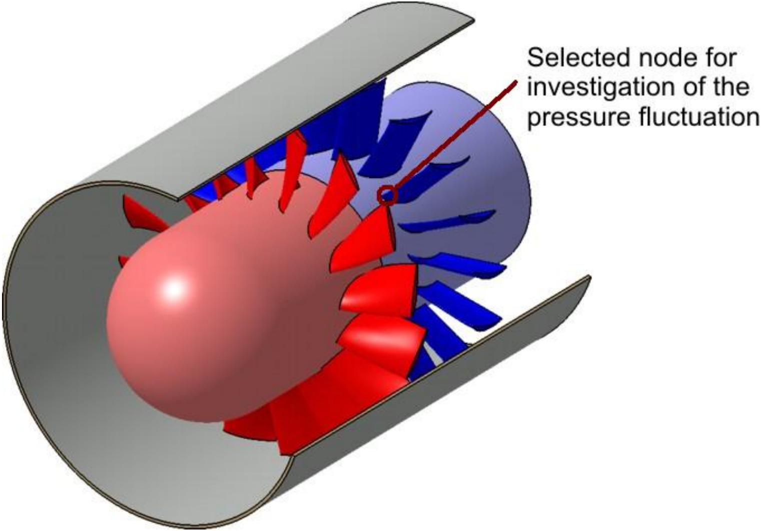
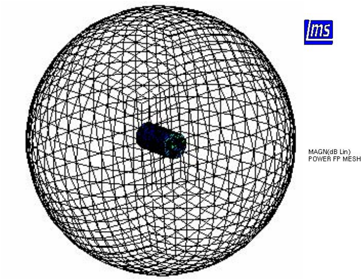
NODE SELECTION FOR CONSIDERATION



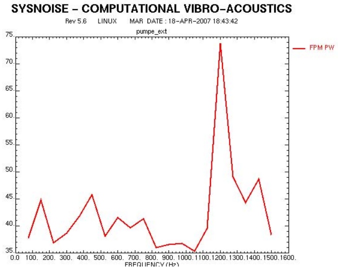
Simulation of the pump

- Evaluation of the dipoles on the stator blades in order to calculate the noise levels on the in- and outlet surfaces
- Analysis over a frequency domain up to 1500 Hz
- Frequency steps $\triangle f = 75$ Hz
- Computation of the acoustic power for a spherical field-point-area with distance $d = 2$ m to the origin

ACOUSTIC POWER



Field-point-area



Acoustic power radiated from the axial pump

Evaluation of the simulation results

- Generally the developed hydrodynamic-acoustic solver is able to predict the expected sound radiation qualitatively good
- But with Lighthill the acoustic power level is computed improperly
- **Reasons:**
  - sound passing through the body of the pump is not computed at all
  - sound radiated from the body is computed inadequately

DRAWBACKS OF LIGHTHILL

- Theory presumes an unresisted sound propagation by means of linear wave equation
- Lighthill analogy acts on the assumption of an unbounded computational domain
- Sound radiation is modeled only into free space

$\Rightarrow$ Effects like reflection, absorption or refraction by solid boundaries can only be considered in combination with discretisation methods (i.e. BEM, FEM)

EIF as alternative option

- Generally accepted in context of small density fluctuations
- Due to the splitting of the viscous and acoustic problem, adapt one grid and integration scheme for
  - solution of the viscous incompressible equations
  - solution of acoustic perturbations

It is supposed that EIF produces more precise predictions of noise generation and propagation than Lighthill does!

- EIF is desired method, which will be pursued in future!

Thank you for your attention!

Direct numerical Simulation

- Solves original 3D unsteady Navier-Stokes equations
- Exact numerical solution of Navier-Stokes equations provides both
  1. fluiddynamic quantities
  2. acoustic quantities

  within the same solution vector

$$\begin{bmatrix} \rho(x_i, t) \\ u_i(x_i, t) \\ p(x_i, t) \end{bmatrix}$$

- Necessity of a numerical mesh with an adequate fine resolution

- Direct computation of NSE with appropriate state equations for 3D unsteady flow demands a grid size with number of mesh points N

$$N \sim Re^3$$

- For technically interesting flows ($Re > 10^5$) DNS needs enormous calculating capacity and calculating time

Return

Derivation of Lighthill's acoustic equation

- Point of departure are the conservation equations

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} &= 0 \\ \rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} &= -\frac{\partial p}{\partial x_i} + f_i \end{aligned}$$

- Multiplying the conservation of mass equation by u_i and adding the product to the momentum equation gives

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + f_i$$

- Taking into account a source term q for the continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = q$$

- Differentiating the mass equation with respect to time, and the momentum equation with respect to x_i results in

$$\begin{aligned}
\frac{\partial^2 \rho}{\partial t^2} + \frac{\partial^2 (\rho u_j)}{\partial t \partial x_j} &= \frac{\partial q}{\partial t} \\
\frac{\partial^2 (\rho u_i)}{\partial x_i \partial t} + \frac{\partial^2 (\rho u_i u_j)}{\partial x_i \partial x_j} &= -\frac{\partial^2 p}{\partial x_i^2} + \frac{\partial f_i}{\partial x_i}
\end{aligned}$$

- By subtracting the latter one from the first equation, one gets the **acoustic wave equation**

$$\frac{\partial^2 \rho}{\partial t^2} - \frac{\partial^2 p}{\partial x_i^2} = \frac{\partial q}{\partial t} - \frac{\partial f_i}{\partial x_i} + \frac{\partial^2}{\partial x_i \partial x_j}\left(\rho u_i u_j\right)$$

- Use a perturbation ansatz for linearisation

$$\begin{aligned} \rho &= \rho_0 + \rho' \\ p &= p_0 + p' \\ u_i &= U_i + u_i' \end{aligned}$$

- U_i is the time-dependent solution, u_i' the turbulence induced fluctuation velocity
- Assume an isentropic state change

$$\frac{\partial p}{\partial \rho} = \frac{\frac{\partial p}{\partial t}}{\frac{\partial \rho}{\partial t}} = a_s^2 = \gamma \cdot R \cdot T, \qquad \text{with}$$

  - γ: heat capacity ratio
  - **R**: gas constant
  - **T**: temperature

- Thus

$$\frac{\partial(p_0 + p')}{\partial(\rho_0 + \rho')} = \frac{\partial p'}{\partial \rho'} = a_s^2 = \gamma \cdot R \cdot T = \frac{\frac{\partial p'}{\partial t}}{\frac{\partial \rho'}{\partial t}}$$

- And with

$$\begin{aligned} \frac{\partial^2 \rho}{\partial t^2} &= \frac{\partial^2(\rho_0 + \rho')}{\partial t^2} = \frac{\partial^2 \rho'}{\partial t^2} = \frac{1}{a_s^2}\frac{\partial^2 p'}{\partial t^2} \\ \frac{\partial^2 p}{\partial t^2} &= \frac{\partial^2(p_0 + p')}{\partial t^2} = \frac{\partial^2 p'}{\partial t^2} \end{aligned}$$

$\Rightarrow$ One gets the **Lighthill wave equation** for the alternating density $\rho'(x_i, t)$

$$\frac{\partial^2 \rho'}{\partial t^2} - a_s^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial q}{\partial t} - \frac{\partial f_i}{\partial x_i} + \frac{\partial^2}{\partial x_i \partial x_j}\left(\rho u_i u_j\right).$$

Splitting of the last term

$$\frac{\partial^2}{\partial x_i \partial x_j}\left(\left(\rho_0+\rho'\right)\left(U_i+u'_i\right)\left(U_j+u'_j\right)\right) =$$

$$\underbrace{\frac{\partial^2}{\partial x_i \partial x_j}\left(\rho_0 U_i U_j + \rho_0 U_i u'_j + \rho_0 u'_i U_j + \rho' U_i U_j + \rho' U_i u'_j + \rho' u'_i U_j\right)}_{\text{Mixed Terms}} +$$

$$+ \underbrace{\frac{\partial^2}{\partial x_i \partial x_j}\left(\underbrace{\rho_0 u'_i u'_j}_{\equiv \tau_{ij}}\right)}_{(1)} + \underbrace{\underbrace{\frac{\partial^2}{\partial x_i \partial x_j}\left(\rho' u'_i u'_j\right)}_{\equiv 0}}_{(2)}$$

1. spatial fluctuations of turbulent normal and shear stresses
2. spatial fluctuations of temporal fluct. momentum forces

FLM

Linearisation of the conservation equations

- First consider the continuity equation

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} &= 0 \\ \Leftrightarrow \frac{\partial \rho}{\partial t} + \rho \frac{\partial u_i}{\partial x_i} + u_i \frac{\partial \rho}{\partial x_i} &= 0 \end{aligned}$$

- use the perturbation ansatz

$$\begin{aligned} u_i &= U_i + u_i', \\ \rho &= \rho_0 + \rho' \end{aligned}$$

- Remember

$$f_i = (\rho_0 + \rho') \cdot u_i' + U_i \cdot \rho',$$

- The continuity equation can be transformed to

$$\frac{\partial \rho_0}{\partial t} + \frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_i}\left(u_i' \cdot (\rho_0 + \rho') + U_i \cdot \rho_0 + U_i \cdot \rho'\right) = 0$$

$$\Leftrightarrow \quad \underbrace{\frac{\partial \rho_0}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho_0 \cdot U_i\right)}_{0} +$$

$$+ \quad \frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_i}\left(\underbrace{u_i' \cdot (\rho_0 + \rho') + U_i \cdot \rho'}_{=f_i}\right) = 0$$

- This results in the **linearised continuity equation** for compressible fluids

$$\frac{\partial \rho'}{\partial t} + \frac{\partial f_i}{\partial x_i} = 0$$

- For simplification consider the momentum equation of the Euler equations

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j + p\delta_{ij}) = 0$$

- Inserting the perturbations, one gets

$$\frac{\partial}{\partial t}\left[(\rho_0 + \rho')(U_i + u_i')\right] +$$
$$\frac{\partial}{\partial x_i}\left[(\rho_0 + \rho')(U_i + u_i')(U_j + u_j') + (P + p')\delta_{ij}\right] = 0$$

- Various transformations give the following expression

$$\frac{\partial}{\partial t}\left(\rho_0 \cdot U_i + f_i\right) +$$
$$\frac{\partial}{\partial x_j}\left((U_j + u_j') \cdot \left[\underbrace{u_i'(\rho_0 + \rho') + U_i\rho'}_{=f_i} + U_i\rho_0\right] + (P + p')\delta_{ij}\right) = 0$$

- which results in the **final acoustic momentum equation**

$$\underbrace{\frac{\partial}{\partial t}(\rho_0 U_i) + \frac{\partial}{\partial x_j}\left(\rho_0 U_i U_j + p'\delta_{ij}\right)}_{\equiv 0} \quad +$$
$$\frac{\partial}{\partial t}(f_i) + \frac{\partial}{\partial x_j}\left((U_j + u_j') \cdot f_i + \rho_0 U_i u_j' + p'\delta_{ij}\right) \quad = \quad 0$$

NS3D

Simulation tool, developed at the **FLM**

- Incompressible and compressible fluid flows
- Stationary and nonstationary
- Integrated turbulence model
- Based on Finite-Volume methods

$\Rightarrow$ Ability to handle complex geometries (especially in turbo machinery)

Return