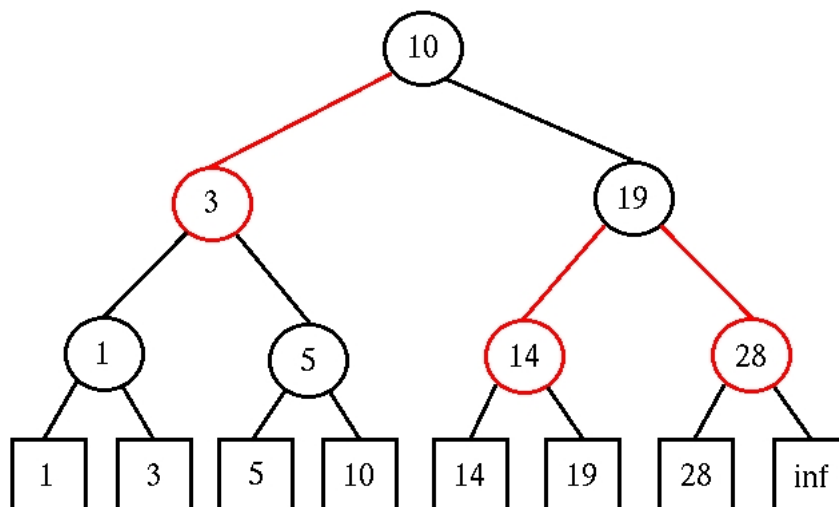

Effiziente Algorithmen und Datenstrukturen I

Aufgabe 1

Consider the following Red-Black-Tree:



Carry out the operations in the following order and show, after each operation, what the Tree looks like (always carry out each operation on the result of the previous operation):

1. insert(7)
2. insert(29)
3. delete(1)
4. delete(14)

Aufgabe 2

Consider a hashtable of size 10 where the first bin is designated as bin 0 (i.e. bins are numbered 0-9). Assuming that our hash function simply returns the last digit of the input (eg. $h(142)=2$, $h(555)=5$) and that we are using a Chaining collision strategy, show what the hashtable looks like after the following series of inserts (you only need to show one table after all inserts):

1. insert(183)
2. insert(6214)
3. insert(3)

4. insert(94)
5. insert(999)

Aufgabe 3

Consider a hashtable of size 10 where the first bin is designated as bin 0 (i.e. bins are numbered 0-9). Assuming that our hash function simply returns the last digit of the input (eg. $h(142)=2$, $h(555)=5$) and that we are using a Linear Probing collision strategy, show what the hashtable looks like after the following series of inserts (you only need to show one table after all inserts):

1. insert(183)
2. insert(6214)
3. insert(3)
4. insert(94)
5. insert(999)

Aufgabe 4

Using the resulting hashtable from the above problem, carry out the following delete operations and show what the hashtable looks like after each operation.

1. delete(6214)
2. delete(183)

Aufgabe 5

Show the expected amortized run-time upper bounds for operations on a dynamic hashtable. Use the potential function given in the notes ($\Phi(s) = 2|w_s/2 - n_s|$, where w_s is the size of the hashtable and n_s is the number of entries in the hashtable) to show that the following holds:

Insert:

$$E[t_{insert}] + \Delta\Phi = O(1)$$

Delete:

$$E[t_{delete}] + \Delta\Phi = O(1)$$