Baseball Elimination

Proof (⇒)

- ▶ Suppose we have a flow that saturates all source edges.
- ▶ We can assume that this flow is integral.
- ▶ For every pairing x-y it defines how many games team x and team y should win.
- ▶ The flow leaving the team-node x can be interpreted as the additional number of wins that team x will obtain.
- ▶ This is less than $M - w_x$ because of capacity constraints.
- ▶ Hence, we found a set of results for the remaining games, such that no team obtains more than M wins in total.
- ▶ Hence, team z is not eliminated.

Project Selection
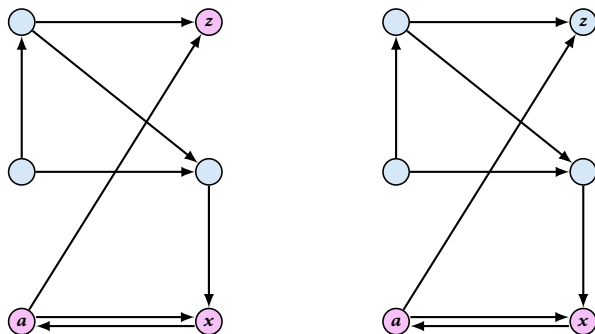
Project selection problem:

- ▶ Set P of possible projects. Project v has an associated profit p_v (can be positive or negative).
- ▶ Some projects have requirements (taking course EA2 requires course EA1).
- ▶ Dependencies are modelled in a graph. Edge (u, v) means "can't do project u without also doing project v."
- ▶ A subset A of projects is feasible if the prerequisites of every project in A also belong to A.

Goal: Find a feasible set of projects that maximizes the profit.

Project Selection

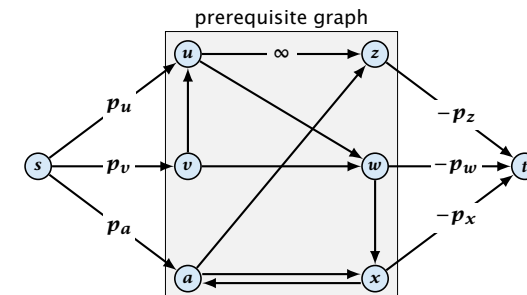**The prerequisite graph:**

- ▶ $\{x, a, z\}$ is a feasible subset.
- ▶ $\{x, a\}$ is infeasible.

Project Selection

Mincut formulation:

- ▶ Edges in the prerequisite graph get infinite capacity.
- ▶ Add edge (s, v) with capacity p_v for nodes v with positive profit.
- ▶ Create edge (v, t) with capacity $-p_v$ for nodes v with negative profit.

prerequisite graph

Theorem 2

A is a mincut if $A \setminus \{s\}$ is the optimal set of projects.

Proof.

- ▶ *A* is feasible because of capacity infinity edges.
- ▶ $\mathrm{cap}(A, V \setminus A) = \displaystyle\sum_{v \in \bar{A}:p_v>0} p_v + \sum_{v \in A:p_v<0} (-p_v) =$

$$\sum_{v:p_v>0} p_v - \sum_{v \in A} p_v$$

For the formula we define $p_s := 0$. Note that minimizing the capacity of the cut $(A, V \setminus A)$ corresponds to maximizing profits of projects in A.

prerequisite graph