

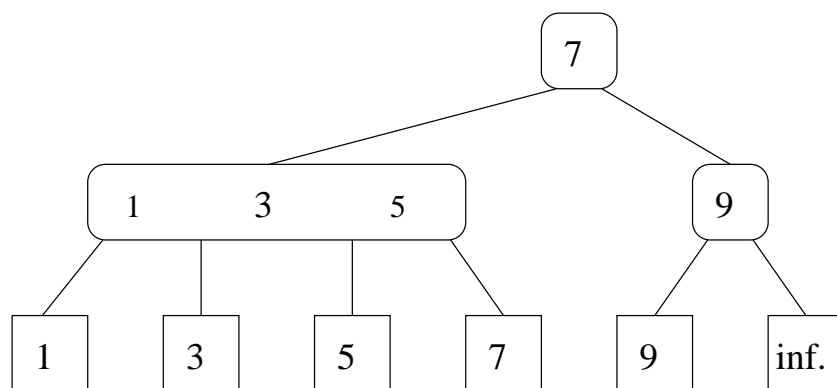
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## Effiziente Algorithmen und Datenstrukturen I

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### Aufgabe 1 (10 Punkte)

Carry out the following operations sequentially on the (2,4) tree shown below so that it remains a (2,4) tree and show what the tree looks like after each operation (always carry out each operation on the result of the previous operation):



1. Insert(4)
2. Delete(3)
3. Delete(1)

### Aufgabe 2 (10 Punkte)

Prove that there exists a sequence of  $n$  insert and delete operations on a (2,3)-tree s.t. the total number of split and merge operations performed is  $\Omega(n \log n)$ .

### Aufgabe 3 (10 Punkte)

Show how to maintain a dynamic set  $Q$  of numbers that supports the operation MIN-GAP, which gives the magnitude of difference of the two closest numbers in  $Q$ . For example, if  $Q = \{1, 5, 9, 15, 18, 22\}$ , then  $\text{MIN-GAP}(Q)$  returns  $18-15=3$ , since 15 and 18 are the two closest numbers in  $Q$ . Make the operations INSERT, DELETE, SEARCH, and MIN-GAP as efficient as possible, and analyze their running times.