Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Winter Semester 2012/13
Problem Set 8
January 8th, 2013

Parallel Algorithms

Due Date: January 15, 2013 **before** *class!*

Problem 1 (10 Points)

How many disjoint s-dimensional hypercubes are contained in an r-dimensional cube for $r \geq s$? (For example, a two-dimensional cube contains two one-dimensional cubes.)

Problem 2 (10 Points)

Given an n-dimensional hypercube, show that the removal of the nodes with size $\lceil \frac{n}{2} \rceil$ and size $\lfloor \frac{n}{2} \rfloor$ results in a bisection containing $\Theta\left(\frac{2^n}{\sqrt{n}}\right)$ nodes.

Problem 3 (10 Points)

Let u and v be nodes of the r-dimensional hypercube, and let $u_1, u_2, \ldots, u_r$ and $v_1, v_2, \ldots, v_r$ denote their neighbors, respectively. Let π be any permutation on $\{1, 2, \ldots, r\}$. Show that there is an automorphism of the hypercube σ such that $\sigma(u) = v$ and $\sigma(u_i) = v_{\pi(i)}$ for $1 \leq i \leq r$.
Hint: An *automorphism* of a graph is a one-to-one mapping of the nodes to the nodes such that edges are mapped to edges.

Problem 4 (10 Points)

Show that any N-node two-dimensional array is a subgraph of the $(\lceil \log N \rceil + 1)$-dimensional hypercube.