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Efficient Algorithms and Datastructures II

Aufgabe 1 (10 Punkte)

Show that the dual of the dual of a Linear Program (in standard form) is the Linear Program itself.

Aufgabe 2 (10 Punkte)

Let $G = (V, E)$ be a bipartite graph. We want to show that the following is an exact LP-relaxation (i.e., always has an integral optimal solution) for the maximum matching problem in G:

$$\begin{array}{lll} \text{maximize} & \sum_e x_e & \\ \text{subject to} & \sum_{e:e \text{ is incident to } v} x_e \leq 1 & \forall v \in V \\ & x_e \geq 0 & \forall e \in E \end{array}$$

An $m \times n$ matrix is said to be totally unimodular if the determinant of every square submatrix of A is +1, -1 or 0. It is known that if A is totally unimodular and b is integral, then the vertices of the polyhedron $P = \{x | Ax \leq b, x \geq 0\}$ are integral. Show that the above linear program is an exact LP-relaxation.

Aufgabe 3 (10 Punkte)

Obtain the dual (D) of the LP above and observe that it finds a fractional minimum vertex cover in G.

Aufgabe 4 (10 Punkte)

Show that the Dual Linear Program (D) above has an integral 0/1 solution. Hence show that Strong Duality implies that the cardinality of a maximum matching equals the cardinality of a minimum vertex cover in a bipartite graph.