Technische Universität München
Fakultät für Informatik
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Problem Set 8
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# Parallel Algorithms

*Due date: January 7th, 2014* **before** *class!*

## Problem 1 (10 Points)

Show how the CCC($d$) is contained in the BF($d$) with wrap-around edges, i.e. can be embedded with a dilation of only 2.

## Problem 2 (10 Points)

Define the *greedy routing* algorithm on a butterfly as seen in the lecture: Every packet crosses the hypercube dimension in increasing order.
In addition, define the *node congestion* to be the highest number of path crossings in any node in the graph. (For node-disjoint routing, the congestion is 1.)
Consider the following two routing problems:

1. A *bit-reversal* permutation maps $x_1x_2\dots x_d$ to $x_dx_{d-1}\dots x_1$.
2. A *transpose* permutation maps $x_1x_2\dots x_d$ to $x_{d/2+1}\dots x_dx_1\dots x_{d/2}$.

Show that the greedy routing algorithm must have a node congestion of $\Omega(\sqrt{n})$ for these two permutation routing problems.

## Problem 3 (10 Points)

Consider a network that consists of two butterflies linked together back-to-front (in contrast to a Beneš network, which consists of two butterflies linked back-to-back). Show how to route the bit-reversal permutation on this network with node congestion 1.

## Problem 4 (10 Points)

Show how to perform any permutation routing problem on a network consisting of four butterflies linked back-to-front with a node congestion of 1.