Technische Universität München
Fakultät für Informatik
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Spring Semester 2014
Problem Set 9
June 23, 2014

Complexity Theory

Due date: June 30, 2014 **before** *class!*

Problem 1 (10 Points)

Show that, if $\mathcal{NP} \subseteq \mathbf{BPP}$, then $\mathbf{RP} = \mathcal{NP}$.

Problem 2 (10 Points)

Show that

1. **RP** and **BPP** are closed under $\preceq_m^p$,
2. **RP** and **BPP** are closed under union and intersection.

Problem 3 (10 Points)

Recall that the definition of **BPP** has the success probability $\frac{2}{3}$. Consider a a definition with success probability $\frac{1}{2}$. The big issue would be the case where there are as many accepting branches as there are rejecting branches. Argue how to alter the PTM to ensue that this case can never occur.

Problem 4 (10 Points)

Error reduction for **RP**: Let $L \subseteq \{0,1\}^*$ be such that there exists a polynomial-time PTM M satisfying for every $x \in \{0,1\}^* : \ x \in L \implies Pr[M(x) = 1] \geq n^{-c}$ and $x \notin L \implies Pr[M(x) = 1] = 0$.
Prove that that for every $d > 0$ there exists a polynomial-time PTM M' such that for every $x \in \{0,1\}^* : \ x \in L \implies Pr[M'(x) = 1] \geq 1 - 2^{-n^d}$ and $x \notin L \implies Pr[M'(x) = 1] = 0$.