Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Assignment 9
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Efficient Algorithms and Datastructures I

Question 1 (10 Points)

(a) Describe how to implement a queue using two stacks and $O(1)$ additional memory, so that the amortized time for any ENQUEUE or DEQUEUE operation is $O(1)$. The only access you have to the stacks is through the standard subroutines PUSH and POP.

(b) A quack is a data structure combining properties of both stacks and queues. It can be viewed as a list of elements written left to right such that 3 operations are possible:

(i) QPUSH: add a new item to the left end of the list

(ii) QPOP: remove the item on the left end of the list

(iii) QPULL: remove the item on the right end of the list

Implement a quack using 3 stacks and $O(1)$ additional memory, so that the amortized time for any QPUSH, QPOP, or QPULL operation is $O(1)$. Again, you are only allowed to access the stacks through the standard functions PUSH and POP.

Question 2 (10 Points)

n motorcyclists $M_1, M_2, \ldots, M_n$ start riding their bikes from a (straight) start line. At the start M_i and M_{i+1} are adjacent to each other. Each motorcyclist M_i starts at some angle ϕ_i and keeps riding in a straight line along this direction at a constant speed $s_i > 0$. Whenever a motorcyclist M_j comes across the path traversed by any other motorcyclist M_i, we say that M_i defeated M_j and in that case, M_j stops riding.

(a) We call the point where M_i defeats M_j as the point of ambush $A_{i,j} \in \mathbb{R}^2$. Show that if $A_{i',j'}$ is a point of ambush which occurs closest to the start line, then i' and j' are consecutive integers.

(b) Show how to enumerate in $O(n \log n)$ time, all events where one motorcyclist defeats another.

Question 3 (5 Points)

For any positive integer n, show a sequence of Fibonacci heap operations that creates a Fibonacci heap consisting of just one tree that is a linear chain of n nodes.

Question 4 (5 Points)

Give a sequence of m MAKESET, UNION and FIND operations, n of which are MAKESET operations, that take $\Omega(m \log n)$ time when we use union by rank only.