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Problem set 4
May 11, 2015
Summer Semester 2015

Online and Approximation Algorithms

Due May 18, 2015 before class!

Exercise 1 (Randomized Ski Rental, Upper Bound - 10 points)

Show that there exists a randomized algorithm for the ski rental problem that achieves a competitive ratio better than 2 against any oblivious adversary.

Exercise 2 (Randomized Ski Rental, Lower Bound - 10 points)

Use Yao's Minimax Principle to prove that the competitive ratio of any randomized algorithm for the ski rental problem is lower bounded by $(1+x)$, where $x \in [\frac{1}{7}, \frac{1}{3}]$ is a constant of your choice.

Exercise 3 (String Scanning, Yao's Minimax Principle - 10 points)

Consider the following problem. Given a string $x \in \{0,1\}^n$, we want to determine if x contains two consecutive 1. By using Yao's MinMax Principle, show that the expected number of bits inspected by any randomized algorithm is $\Omega(n)$.

Exercise 4 (RMTF_p - 10 points)

In the lecture we saw the randomized online list update algorithm RMTF, that moves the requested element to the front of the list with probability $\frac{1}{2}$. We consider a generalized version RMTF_p that moves a requested element to the front of the list with probability $p \in (0,1)$. Show that the competitive ratio of RMTF_p is lower bounded by $\frac{1}{p} - \epsilon$ for any constant $\epsilon > 0$.