## Fundamental Algorithms

## Exercise 1

HOMECOMPUTER shall be a machine that can perform $10^{9}$ operations per second. Consider that we have five different algorithms for a specific problem. For each algorithm $i$, we know the number of operations $T_{i}(n)$ it will perform on a problem of size $n$ :

$$
\begin{aligned}
& T_{1}(n)=6000000 \cdot n \\
& T_{2}(n)=60000 \cdot n \log n \\
& T_{3}(n)=0.003 \cdot n^{2} \\
& T_{4}(n)=10^{-6} \cdot n^{3} \\
& T_{5}(n)=10^{-18} \cdot 2^{n}
\end{aligned}
$$

For each algorithm compute the size $n_{\max }$ of the largest problem the respective algorithm can solve within 1 second ( 1 minute, 1 hour, ...). Enter the maximal problem sizes into the following table:

|  | 1 s | $1 \mathrm{~m}(60 \mathrm{~s})$ | $1 \mathrm{~h}(3600 \mathrm{~s})$ | $1 \mathrm{~d}(86400 \mathrm{~s})$ | $30 \mathrm{~d}\left(\approx 2.6 \cdot 10^{6} \mathrm{~s}\right)$ | $1 \mathrm{a}\left(\approx 3.2 \cdot 10^{7} \mathrm{~s}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T_{1}$ |  |  |  |  |  |  |
| $T_{2}$ |  |  |  |  |  |  |
| $T_{3}$ |  |  |  |  |  |  |
| $T_{4}$ |  |  |  |  |  |  |
| $T_{5}$ |  |  |  |  |  |  |

## K-Exercise 2 (MergeSort)

Compute the number of comparisons that will be performed by MergeSort in the best case (i.e., compute this number exactly, including the constants for $\Theta(n \log n)$.

## Note:

A K-Exercise, in this course, will mark an exercise that might (in a similar form) well occur in the exam.

## Exercise 3 (Sorting)

Prove or disprove the following statement - or try at least to figure out whether it holds or not: If we sort each row of a matrix, and, after that, sort each column of the matrix, the rows of the matrix will still be sorted afterwards.

